Unsteady Flow Phenomena in the Near Wake of a Circular Cylinder

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Abstract

RECENT experimental investigations of incompressible flow around a circular cylinder have indicated the existence of highly complex secondary phenomena near the separation point. In particular, the formation of secondary vortices in the Reynolds number range of 800–5000 and an unsteady flow instability at higher Reynolds numbers have been observed. The familiar vorticity/stream-function formulations used to study these phenomena typically employ ADI for the parabolic vorticity transport equation and an iterative method for the elliptic stream-function equation. In the present approach, a second-order fast Poisson solver based on the Fast Fourier transform is used instead of an iterative technique in order to increase overall solution accuracy and significantly reduce computation times. The numerical results indicate good agreement with experiment and other higher-order solution techniques. A study of different initial conditions shows that they can influence the flow development to some extent. Consequently, these conditions must be chosen correctly if one is to maintain accuracy in time.

Contents

Separated flows and the mechanisms of vortex generation are of great interest in aerodynamics. Recently, illuminating experimental work by Bouard and Coutanceau[1] has documented the existence of important and complicated secondary phenomena in the wake and near the separation point of a circular cylinder in incompressible flow. For Reynolds numbers (based on the diameter of the cylinder) between 100 and 400, they observe near the surface a bulge in the streamline pattern that first occurs about halfway between the separation point and the rearward stagnation point. For higher Reynolds numbers, up to about 800, this bulge becomes a closed set of streamlines and constitutes a secondary vortex. The rotation of the vortex is opposite to that of the main wake flow and is, in fact, due to the separation of the backflow itself. Such a secondary vortex is identified by negative surface vorticity in the wake region, since the fluid rotation is in the counterclockwise sense.

A recent numerical approach to the simulation of incompressible viscous flow using the Navier-Stokes equations written in vorticity/stream-function form has been described by Loc.[2] This method was later used by Loc and Bouard[3] to examine the flow behavior described above. The method employs an ADI procedure for the vorticity transport equation in conservation form while relying on a compact fourth-order Hermitian finite-difference formulation for the Poisson equation. The compactness is achieved by introducing the second derivatives of the stream function as additional unknowns in the problem. This results in a block tridiagonal matrix system that must be solved iteratively at each time step. A number of different Reynolds number cases were computed and general promise for the method was demonstrated.

The present approach retains the use of ADI for the convective and diffusive transport of vorticity, but employs an alternative treatment for the Poisson equation. For certain problems, it has long been realized that fast direct methods for the solution of separable elliptic equations are more efficient and accurate than iterative techniques. However, such direct solvers, often based on the Fast Fourier transform (FFT), are commonly believed to be too cumbersome to code. Since the introduction of the generalized FFT's and the general availability of user-friendly subroutines for their computation,[4] it is now just as easy to code a fast Poisson solver as it is to program ADI. Furthermore, the generalization of the technique has removed the previous restrictions on the number of mesh points allowed in each direction. The amount of storage required is little more than ADI and we may thus choose the size of the grid as if we were going to use an iterative method.

The purpose of the present study is thus twofold. First, it is desired to compare the results obtained with the present second-order fast Poisson solver to those obtained with Loc's fourth-order iterative method using the same computational parameters. This comparison will allow an evaluation of the consequences inherent in trading off accuracy for speed. Second, it is of interest to investigate the influence of the initial conditions on the transient flow solution. Although one normally begins the computation by impulsively starting the cylinder from rest, it is possible to simulate a gradual acceleration to the final velocity. The determination of the relative importance of flow history in the transient phase may provide some useful information regarding the time accuracy of the solutions.

Governing Equations

The two-dimensional incompressible Navier-Stokes equations in vorticity/stream-function form may be written as

$$\frac{\partial \zeta}{\partial t} + \mathbf{V}\cdot\nabla\zeta = \frac{2}{Re}\nabla^2\zeta \tag{1}$$

and

$$\nabla^2\psi = -\zeta \tag{2}$$

where ζ is the vorticity and ψ the stream function. The variables have been nondimensionalized with respect to freestream quantities and the radius of the cylinder a. The Reynolds number is based on the cylinder diameter and is defined as $Re = \rho_\infty V_\infty (2a)/\mu_\infty$. The boundary conditions required are the no-slip condition at the solid surface (which is also a streamline), potential flow at the outer boundary, and suitable symmetry conditions at appropriate boundaries (only half of the cylinder is considered). These requirements are enough to completely determine ψ and ζ within the domain.

The finite-difference implementation of Eqs. (1) and (2) is facilitated if we consider a transformation from polar (r,θ) to generalized (ξ,η) coordinates such that $r = r(\xi)$ and $\theta = \theta(\eta)$.

Received Nov. 19, 1986; Presented as Paper 87-0371 at the AIAA 26th Aerospace Science Meeting, Reno, NV, Jan. 12–15, 1987; synoptic received May 15, 1987. Full paper available from AIAA Library, 555 W. 57th St., New York, NY 10019. Price: microfiche, $4.00; hard copy, $9.00. **Remittance must accompany order.**

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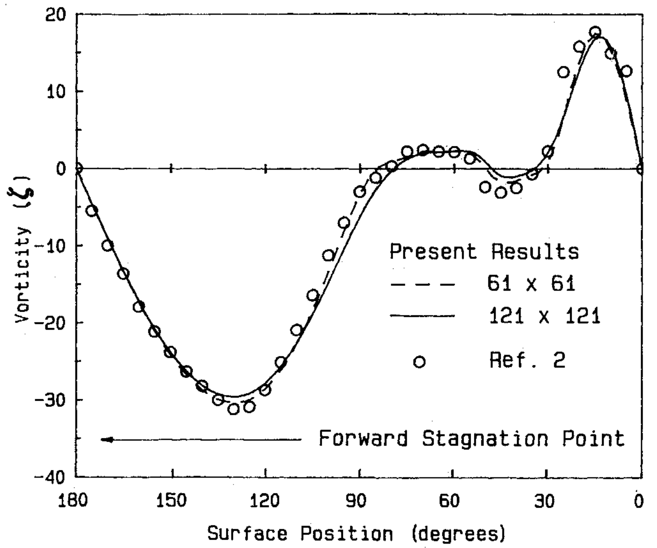

Fig. 1 Comparison of the surface vorticity with the results of a fourth-order Poisson solver using the same grid at $t=5$ and $Re=550$.

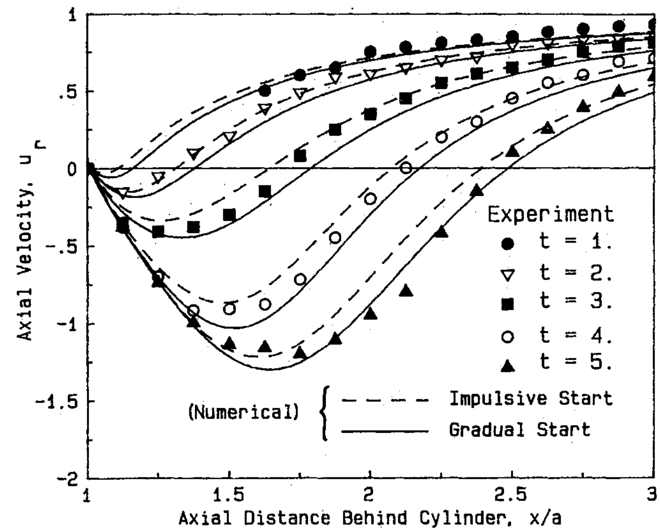

Fig. 2 Influence of the initial conditions on the development of the axial velocity profiles in the wake, as compared to an experiment for $Re=3000$.

Under this transformation, given analytically as $r=e^{\pi\xi}$ and $\theta=\pi\eta$, we obtain

$$\pi^2 r^2 \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial \xi}(u\zeta) + \frac{\partial}{\partial \eta}(v\zeta) = \frac{2}{Re}\nabla^2\zeta \tag{3}$$

and

$$\nabla^2\psi = -\pi^2 r^2 \zeta \tag{4}$$

where

$$\zeta = \frac{\partial v}{\partial \xi} - \frac{\partial u}{\partial \eta} \tag{5}$$

$$u = \frac{\partial \psi}{\partial \eta} \tag{6}$$

$$v = -\frac{\partial \psi}{\partial \xi} \tag{7}$$

The quantities u and v are related to the r and θ velocity components in the physical plane by

$$u = \pi r u_r \tag{8}$$

$$v = \pi r u_\theta \tag{9}$$

Since the Laplacian operator in the computational plane retains the familiar form $\nabla^2 = \partial^2/\partial\xi^2 + \partial^2/\partial\eta^2$, this transformation has very nearly made a cylindrical problem into a Cartesian problem, thus considerably simplifying the numerical implementation.

The limits for ξ and η are $0 \le \xi \le \xi_{max}$ and $0 \le \eta \le 1$. The body surface is located at $\xi=0$ and the outer boundary at $\xi=\xi_{max}$. Since only half of the cylinder is considered, η ranges from 0 at the rearward stagnation point to 1 at the forward stagnation point. In this problem, the exact potential flow solution is used at the outer boundary, so that

$$\psi_\infty = 2 \sinh \pi\xi_{max} \sin \pi\eta \tag{10}$$

Results

For the first calculation, the computational parameters are chosen so that a direct comparison with the fourth-order results of Ref. 2 can be made. Specifically, a 61×61 grid is used with $\xi_{max} = 0.954$, $\Delta t = 0.033$, and $Re = 550$. The surface vorticity at $t=5$ is compared in Fig. 1, and the agreement is quite reasonable. A second solution with the grid refined to 121×121 nodes offers no significant improvement. The trends of all three solutions are similar, including the development of a secondary vortex observed between about 30 and 45 deg from the rearward stagnation point. In this case, it appears that the exact numerical solution obtained using a second-order direct method for the Poisson equation differs little in detail from that obtained using a fourth-order iterative procedure.

Another unsteady feature of interest in the wake is the development of the axial velocity u_r, shown compared to an experiment[1] in Fig. 2. The grid used in this case is 79×129, with $\xi_{max}=0.704$, $\Delta t=0.02$, and $Re=3000$. The two initial conditions considered are an impulsive start and a gradual start in which the cylinder is accelerated over 15 time steps. The gradual starting procedure is often helpful in reducing the severity of high-frequency transients that occur in high Re calculations. The impulsive start, however, which conforms to the way the experiment was conducted, produces better agreement with the data for $t \le 3$. For later times, one could argue for either solution. Nonetheless, it would appear best to follow the experimental conditions as much as possible.

References

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